# Mat 2377: Quiz #5

July 20, 2016

Answer all the questions first in one 45 minute sitting. Then check your answers against the solutions given below.

1. Suppose we have a random sample $X_1, ..., X_{15}$ of size 15 from the normal distribution.

a) Obtain a point estimate of the population mean and for the population variance

b) calculate a 95% confidence interval for the population mean based on the following data

31.5 36.9 33.8 30.1 33.9 35.2 29.6 34.4 30.5 34.2 31.6 36.7 35.8 34.5 32.7

c) calculate a 90% confidence interval for the data in b)

2. Calculate a 95% confidence interval for the variance in question 1

3. The length of life of a type A light bulb is normally distributed with mean $\mu_X$ and variance 784 whereas for a type B light bulb it is normally distributed with mean $\mu_Y$ and variance 627. We take a sample of 56 type A and 57 type B. We observe $\bar{x} = 937.4$ hours and $\bar{y} = 988.9$ hours. Find a 90% confidence interval for the difference $\mu_X$-$\mu_Y$.

4. How lare a sample would be needed to obtain a 95% confidence interval of width at most 1 for the mean of a normal whose variance is equal to 2.25?

5. Suppose that we wish to compare the weight of calcium in standard

cement and cement doped with lead. Reduced levels of calciumwould imply that water would attack various locations in the cement structure. Assuming normal distributions calculate a 95% confidence for the difference $\mu_X - \mu_Y$ in population means with the following information. Assume a common variance for the two distributions.

| Section | sample mean | sample variance | sample size |
|---|---|---|---|
| Standard cement ($X$) | 90 | 25 | 10 |
| lead doped cement ($Y$) | 87 | 16 | 15 |

6. A political candidate discovers that 185 out of 351 voters are in favor of him. Construct a 95% confidence interval for the fraction of the voting population in his favor.

7. A telephone poll of 800 adult Americans of whom 374 were gun owners and 426 did not own guns showed that 206 gun owners and 338 non gun owners favored stricter gun-control laws. Construct a 95 % confidence interval for the difference in proportions between gun owners and non gun owners

Solutions

1. We first note that the mean and variance of the normal are both unknown. Point estimates are respectively

a) $\bar{x} = 33.427, s^2 = 5.462$

b) We must use the Student t distribution with $n-1 = 14$ degrees of freedom

A 95% confidence interval is $x \pm \bar{t}\sqrt{\frac{s^2}{n}}$Here,

$t = 2.145$and hence the interval is $(32.13, 34.72)$

c) Similarly, a 90% confidence interval is $(32.36, 34.49)t = 1.761$

2. For the variance, we must use the Chi square distribution with $n-1 = 14$ degrees of freedom. We look up

$$\chi^2_{0.025} = 26.119, \chi^2_{0.975} = 5.629$$

Hence the 95% confidence interval is

$$[\frac{(n-1)\,s^2}{\chi^2_{0.025}}, \frac{(n-1)\,s^2}{\chi^2_{0.975}}] = (2.93, 13.59)$$

3. Since the variances are known, we use the two sample confidence interval based on the normal distribution. It is given by

$$\bar{x} - \bar{y} \pm z_{\alpha/2}\sqrt{\frac{\sigma^2_X}{n_1} + \frac{\sigma^2_Y}{n_2}}$$

$$937.4 - 988.9 \pm 1.96\sqrt{\frac{784}{56} + \frac{627}{57}}$$

$$(-61.3, -41.7)$$

4. The sample size required must satisfy the equation

$$\begin{aligned} n &= \left(\frac{2\sigma z_{\alpha/2}}{w}\right)^2 \\ &= \left(\frac{2\,(1.5)\,1.96}{1}\right)^2 \\ &= 34.57 \approx 35 \end{aligned}$$

5. The pooled estimate of common variance is equal to

$$\begin{aligned} s^2 &= \frac{(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2}{n_1+n_2-2} \\ &= \frac{(9)\,25 + (14)\,16}{23} \\ &= 19.52 \end{aligned}$$

We must use the Student t distribution with 23 degrees of freedom. Note $t_{0.025} = 2.069$

Hence the 95% confidence interval is given by

$$\bar{x} - \bar{y} \pm t_{0.025} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$90 - 87 \pm 2.069\,(4.4)\,\sqrt{\frac{1}{10} + \frac{1}{15}}$$

$$(-0.72, 6.72)$$

The interval contains 0 and so we have no evidence that the two types of cement differ at that level of confidence.

6. This interval is given approximately by

$$p \pm z_{\alpha/2}\sqrt{\frac{p\,(1-p)}{n}}$$

where $p = \frac{183}{351}$. This gives the interval $(0.475, 0.579)$

7. This interval is given approximately by

$$p_1 - p_2 \pm z_{\alpha/2}\sqrt{\frac{p_1\,(1-p_1)}{n_1} + \frac{p_2\,(1-p_2)}{n_2}}$$

where $p_1 = \frac{206}{374}, p_2 = \frac{338}{426}$. This gives the interval $(-0.306, -0.179)$

The interval contains 0 and hence we conclude there is a significant difference between the two groups. Since $P_1 - P_2 < 0$, we see that the gun owners are less in favor of stricter gun control laws.